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RAN-2203000205023006**T.Y.B.Sc. (Sem. V) Examination November - 2023****Mathematics : MTH-506 :****Number Theory - I****Time: 2 Hours]****[Total Marks: 50****सूचना : / Instructions**

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नीचे दशविवेक निशानीवाणी विगतो उत्तरवली पर अवश्य लपववी.

Fill up strictly the details of signs on your answer book

Name of the Examination:

T.Y.B.Sc. (Sem. V)

Name of the Subject :

Mathematics : MTH-506
Number Theory - I

Subject Code No.: 2203000205023006

Seat No.:

Student's Signature

- (2) All questions are compulsory.
- (3) Follow usual notations.
- (4) Figures to the right indicate marks of the question.

Que 1 Answer Five any of the following:**(10)**

- (1) If $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$ then prove that $\gcd(a, bc) = 1$.
- (2) Prove that square of any integer is of the form $3k$ or $3k + 1$.
- (3) For any arbitrary integer a , verify that $2|a(a + 1)$.
- (4) For any integer a , prove that $a^4 = 0$ or $1 \pmod{5}$.
- (5) Assuming that $\gcd(a, b) = 1$ then prove that $\gcd(2a + b, a + 2b) = 1$ or 3 .
- (6) Verify whether integers 4, 11, 12, 13, 22, 82, 91 form a CRS $\pmod{7}$.
- (7) Prove that the only prime of the form $n^3 - 1$ is 7.
- (8) $2^4 \equiv \underline{\hspace{2cm}} \pmod{7}$.

Que 2 Answer any two questions: (10)

- (1) Prove that the square of any odd integer is of the form $8k + 1$.
- (2) Find integers x and y such that $\gcd(119, 272) = 119x + 272y$.
- (3) For positive integers a and b prove that $\gcd(a, b) \operatorname{lcm}(a, b) = ab$.

Que 3 Answer any two questions: (10)

- (1) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$ where $d = \gcd(a, b)$. If $x = x_0, y = y_0$ is a particular solution then find all other solutions.
- (2) Determine all the positive solutions in the integers of the Diophantine equation $18x + 5y = 48$.
- (3) If $p \geq q \geq 5$, p and q both are prime then prove that $24 \mid p^2 - q^2$.

Que 4 Answer any two questions: (10)

- (1) Show that $89 \mid 2^{44} - 1$ and $97 \mid 2^{48} - 1$.
- (2) If $p \geq 5$ is a prime then show that $p^2 + 2$ is composite.
- (3) Prove that for any composite integer a will always have a prime divisor p such that $p \leq \sqrt{a}$.

Que 5: Answer any two questions: (10)

- (1) Prove that $111^{333} + 333^{111} \equiv 0 \pmod{7}$.
- (2) Without performing the division, determine whether the integer 738969 is divisible by 9 and 11 or not.
- (3) State and prove the Divisibility Test for 11.
